Probability and strategy in a variation of a blackjack game

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Abstract

The subject of this paper is a variation of a blackjack game, mainly popular in some parts of Europe where it is known as einz (in German slang: one). We describe the rules of this game, indicate its main characteristics, give some probabilities, suggest the strategy, and consider some typical situations.

1 Introduction

Einz is a game for two or more players played with one or more decks of 52 cards. Face cards king, queen, and jack are counted as 4, 3, and 2 points, respectively. An ace is counted as 11 points, and the remaining cards are counted as the numeric value shown on the card. The value of a hand is simply the sum of the point counts of each card in the hand. Any hand with value of 21 is called an *einz*, as well as the hand with exactly two cards and value of 22 (these cards must be two aces). The object of the game is to reach an einz or to reach a score higher than the opponents without exceeding 21. Scoring higher than einz (a *busting*) results in a loss.

There are two basic types of this game:

(1) game with no dealer (or *open game*) in which a player is playing against all other players, and

(2) game with a dealer (or *dealer game*) in which a player is playing against the dealer, but not against any other players.

In an open game, the cards are dealt by a different player in each round. The player who is dealing the cards does not have any special status in relation to the others. At begin, every player is dealt an initial hand of two cards. Then, each player, one by one, is getting the additional cards in order to achieve his total score. All cards in the hand are visible to himself only, and a player can stand at any time. A player must announce a bust immediately after it happens, and in this case he lose automatically. If a player gets an einz, he is showing his hand, and he wins the round (no matter if any other players did not got a chance to make their total scores). If none player got an einz, the winner is whoever has closest to a total of 21.

There is an additional rule usually known as *changing on 14*: each player can start over with getting the new cards whenever his total score equals 14. If he decide to get the new cards, he is showing the hand, and continue with the new one.

There are few variations of a dealer game, and they will be explained and discussed in Section 3.

There are many, more or less popular, variations of the blackjack game. The main characteristics of the einz game are the distinct values of face cards, immediate announcing of a bust or an einz, the einz with 2 aces, and changing on 14. Note that some of these rules appear in some other variations of the game. On the other hand, a very common rules of many popular blackjack games (splitting the hand, soft and hard ace, etc – see [1, 2]) are not present here.

The strategy in usual blackjack and its simple variations (played in worldwide casinos) is well developed. It includes basic probabilistic methods, as well as so called card counting [1, 2, 3] - a card game strategy used to determine whether the next hand is likely to give a probable advantage to the player or to the dealer. The most common variations of card counting are based on statistical evidence that high cards, especially aces and all cards counted as 10 points (there, face cards are all counted as 10 points, as well as 10s), benefit the player more than the dealer, while the low cards, especially 4s, 5s, and 6s, help the dealer while hurting the player. Thus, a high concentration of high cards in the deck increases the player's chance, while low cards benefit the dealer. Contrary to the usual blackjack, in einz we have more low than high cards, and the usual counting principles cannot be transferred. But in any case, here we consider the einz as an easy relaxing game, and therefore through the whole paper we will not include the counting strategy nor playing for the money. In order to avoid the possibility of counting, we will assume that each round starts with a full set of cards.

In the following two sections we consider the open and the dealer game, respectively. Some typical situations are discussed in the last section. We use the usual terminology, and techniques in probability that can be found in any complete book concerning this discipline, say [4] or [5].

2 Open game

2.1 When to stand?

Here we consider the question: when the player should stand in the open einz game? According to the rules, two obvious candidates for the standing strategy arise

(1) the player should stand when its score is at least 17, or

(2) the player should stand when its score is at least 18.

If the player uses the standing strategy (1) in every round, i.e. if he is taking the cards until his hand busts or achieves a score of 17 or higher in every round, we will say that "he always stands on 17". Similarly, if he uses the strategy (2), we will say that "he always stands on 18".

We compute the probabilities of all possible outcomes in both suggested strategies. To simplify the computation, we assume that the player never changes on 14. An einz in two cards can be achieved if these two cards are either both aces or an ace and a ten. Therefore, the probability that the player gets an einz in two cards is easily computed and it is equal to 0.016. The remaining situations are resolved in the similar way (we use the computer to complete the whole computation procedure). The results are given in Table 1 and Table 2.

♯ cards / score	17	18	19	20	einz
2	0.036	0.027	0.024	0.016	0.016
3	0.074	0.067	0.051	0.035	0.038
4	0.040	0.042	0.056	0.036	0.027
5	0.010	0.009	0.013	0.012	0.008
> 5	< 0.001	< 0.001	< 0.002	< 0.002	< 0.002
any	0.161	0.147	0.145	0.100	0.092

Table 1: Probabilities of scores between 17 and einz for the player who always stands on 17.

♯ cards / score	18	19	20	einz
2	0.027	0.024	0.016	0.016
3	0.067	0.056	0.040	0.044
4	0.042	0.066	0.046	0.038
5	0.009	0.018	0.017	0.014
> 5	< 0.001	< 0.003	< 0.003	< 0.003
any	0.147	0.167	0.122	0.114

Table 2: Probabilities of scores between 18 and einz for the player who always stands on 18.

By summation of the values in the last row of Table 1, we get that the probability of score 17 or better in a single hand is equal to 0.645, and thus the probability of busting is 0.355. Considering Table 2, we get that these two probabilities are in this case equal to 0.550 and 0.450, respectively. In the first look the first strategy seems to be better, but let us consider both of them in a real game with two players.

Assume first that both players always stand on 17. Then, according to the rules, the first player wins in the following situations: (a) if he gets an einz, (b) if his score is between 17 and 20, and the second player busts, (c) if his score is between 17 and 20 and it is higher than score of the second player. Using Table 1, we get the probabilities of all these outcomes: (a) 0.092, (b) 0.196, and (c) 0.114, which together give the first player's winning probability equal to 0.402. The round is tied if both players have equal score between 17 and 20, the probability of that is 0.079. Finally the second player's winning probability is equal to 1-0.402-0.079=0.520. These results are given in the first row of Table 3. In the same way we get the remaining data of the same table.

result / stand on	17 vs 17	17 vs 18	18 vs 17	18 vs 18
player 1 wins	0.402	0.379	0.399	0.373
tied	0.079	0.058	0.058	0.064
player 2 wins	0.520	0.563	0.543	0.562

Table 3: Probabilities in the game with two players with given strategies.

Regarding Table 3, the strategy (2), i.e. "always stand on 18", is clearly better choice for the second player, while for the first player the strategy (1) has a tiny advantage.

The game with more than 2 players can be considered in the similar way. Here we give just one result: in the game with 3 players such that the first two players always stand on 17, and the third player always stand on 18, the first player's winning probability is 0.1966, the second's is 0.1881, and the third's is 0.3494, while at least two players have mutually equal winning score with probability of 0.2659. In this game the first player has a bit better chances than the second one (that is because the round is over if the first player gets an einz, while if he busts, the second player can bust as well with equal probability). Note also that the third player has a clear advantage, and the same holds in any game with k ($k > 3$) players: the k-th player has advantage against any other. Observe finally that the game with 3 players reduces to a game with 2 players (along with the probabilities presented above) whenever the first player busts.

Conclusion: The first player should stand on 17, the second player should stand on 18. In the game with more than two players, the last player has a clear advantage.

2.2 Change on 14 or not?

We consider the question in the title of this part. In order to get the exact probabilities we consider an einz game with 52 cards, and we also assume that the player gets 10 and 4 in his first two cards (the conclusion given in the end is the same in any other case, i.e. it does not depend on these assumptions).

Assume first that the player always stands on 17. Then he does not bust if his next card is counted as 3, 4, 5, 6, or 7 (recall that queens are counted as 3, as well as 3s), or if his next two cards are counted as 2 and k ($k = 2, 3, 4, 5$) and if they are taken in the given order.

We compute the probabilities: $P(3) = \frac{8}{50}, P(4) = \frac{7}{50}, P(5) = P(6) = P(7) = \frac{4}{50}$, and $P(2,2) = \frac{56}{2500}, P(2,3) = \frac{64}{2500}, P(2,4) = \frac{56}{2500}, P(2,5) = \frac{32}{2500}$. Altogether, the probability of 17 or better score is equal to the sum of these probabilities and it is equal to 0.624, i.e. it is less than the probability of the same result if he changes on 14 (this probability is slightly different from 0.645 (this results is obtained in the previous subsection) since two cards, 10 and 4, are removed from the deck, but it is very close to this number). Therefore, change on 14 is a bit better choice for this player.

Assume now that the player always stands on 18. If he decides to get another one or two cards, we get that the probability of 18 or better score is equal to 0.514, i.e. it is again less than the probability of the same score if he changes on 14.

Conclusion: Regardless of his standing strategy, a player will tiny increase his chances if he changes on 14.

2.3 Probability and mathematical expectation of possible scores

What we can say about his hand if a player stands after he took a few cards? Clearly, he did not busted and he did not got an einz (since both results must be announced), and so he has some high score. It is an intriguing question: which score he probably has? In other words, it would be interesting to compute the probabilities of all possible scores. This computation depends on (earlier discussed) playes's standing strategy, and the number of cards that he took.

Assume that the player always stands on 17, and that he stands after he took 2 cards. Then, according to the first row of Table 1, the probability that he has 17 is equal to $0.036/(0.036 + 0.027 + 0.024 + 0.016) = 0.350$. In the similar way, we compute the remaining probabilities (only if he took between 2 and 5 cards, the remaining situations are very rear), and the results are given in the first part of Table 4. In the second part of this table we give the results for the player who always stands on 18.

♯ cards / score	17	18	19	20
2	0.350	0.262	0.233	0.156
3	0.326	0.295	0.225	0.154
4	0.230	0.241	0.322	0.207
5	0.227	0.205	0.295	0.273
2		0.403	0.358	0.239
3		0.411	0.344	0.245
4		0.273	0.429	0.299
5		0.205	0.409	0.386

Table 4: Probability of score if the player stands with k ($k = 2, \ldots, 5$) cards in the hand.

As we can see, if the player took only two cards, there are small chances that he has the highest possible score (i.e. 20), but these chances increase with the number of cards. The remaining possible scores can be analyzed in the similar way. Another application of this computation is given below.

Assume, for example, that we have a game with two players, and let say that we know that both of them always stand on 17 (different possibilities can be considered in the similar way). If both of them stand after they took equal number of cards then their chances to win are also equal (nobody got an einz and nobody busted). What if they took a different number of cards? The probabilities of the possible results are given in Table 5. There, regardless of the number of cards, the probability of tied round is about 0.25 (i.e. 1 of 4 rounds should be tied). On the other hand there is a large difference in wining probabilities.

result / ♯ cards	2 vs 3	2 vs 4	2 vs 5	3 vs 4	3 vs 5	4 vs 5
player 1 wins	0.361	0.293	0.273	0.296	0.276	0.344
tied	0.268	0.251	0.244	0.250	0.243	0.253
player 2 wins	0.371	0.456	0.483	0.454	0.481	0.403

Table 5: Probabilities in the game with two players (both always stand on 17) if they achieved their scores between 17 and 20, the first with k, and the second with l cards ($2 \leq k < l \leq 5$).

The mathematical expectation (if necessary, see definition in [4, Chapter 6]), or in the case of this game, the expected average score of the player who stands after he took 2 cards (if he always stands on

17) is (compare the first row of Table 4) $E(2; 17-20) = 0.350 \cdot 17 + 0.262 \cdot 18 + 0.230 \cdot 19 + 0.227 \cdot 20 = 18.156$. The remaining mathematical expectations (computed including or not including an einz into the high score) in the case of two standing strategies are given in Table 6.

E / ♯ cards	2	3	4	5	any
17-20	18.156	18.207	18.506	18.614	18.332
17-einz	18.571	18.608	18.841	18.981	18.707
18-20	18.836	18.834	19.026	19.182	18.943
18-einz	19.256	19.295	19.417	19.621	19.369

Table 6: Average value of the high score achieved with k ($2 \leq k \leq 5$) cards.

Conclusion: When a player stands the following rule can be applied: more cards in his hand - better result can be expected. The average score also increases in the number of cards.

3 Dealer game

In this type of the game the dealer is dealing the cards in every round, and therefore his hand is resolved after all players have finished playing. In many variations of the blackjack game the dealer's playing is strictly determined by certain rules directing his every move in the game. Commonly, the dealer game in einz is less popular than the open game, but here we present a several possibilities for the dealer's playing describing at the same time the whole game.

First, it is usual that the additional possibility, changing on 14, is not allowed to the dealer (the same holds for the similar rules in better known variations of this game (like "splitting the hand" [1, 2])). It is also usual that all cards dealt are visible to everyone.

The possibilities of the remaining rules follow:

(i) Here, contrary to the open game the busting and getting an einz do not mean automatic lose or win. In this case, if the dealer equals the player's result (both busted, or got the equal score including the einz) the game is tied. The dealer always stands on, say, 17. This variation is very similar to many of popular types of the blackjack game. But, without possibility that anyone changes on 14, these rules give the equal chances to a player and dealer whenever the player applies the dealer's standing strategy. If the player is allowed to use the opportunity to change on 14, then his winning probability is a bit higher, which is not usual, and would be corrected by some additional rule concerning the paying the wins (this part of the game is beyond the subject of this paper).

(ii) Regarding a player, all rules of the open game remain unchanged. The dealer always stand on 17. By the rules of this variation, the dealer wins if (a) the player busts, (b) the dealer's score is strictly higher than the player's score.

According to these rules, none round is tied. If the player always stands on 17, the probability that he busts is 0.355, and if he does not bust the probability that the dealer gets a higher score is 0.165. Thus, the player's winning probability is 0.480 (against the dealer's 0.52) – see Table 3. Using the opportunity to change on 14, and by changing his standing strategy, the player can increase his chances (but not over 0.5). Since, it is usual that the dealer has a better chances, but close to 0.5, this variation of dealer game is the author's recommendation to be used.

If the player always stands on at least 18, his winning probability is 0.458 (less than the above).

(iii) The same as (ii), with exception that the dealer always stands on 17.

In this case the dealer's chances are better than the above. Regardless of player's standing strategy, the dealer's winning probability is about 0.563.

4 Typical situations

There are many typical situations in both open and dealer game that can be resolved using results of Section 2. We consider just few of them.

1. *Open game with 8 card decks. Three players are playing, the first two both stand with 2 cards in the hand, while the third scored 18 in his 2 cards. What should he do?*

   Every player took 2 cards, and the value of each of these 6 cards is higher than 5 (if we naturally assume that none player stands before 17). Assume first that the first two players both always stand on 17. If the third player stands then, according to the simple computation based on Table 1, his chances are about these numbers:

   - win: 0.123
   - win together with one player: 0.104
   - all 3 players tied: 0.069
   - lose: 0.705

   If he take another card, he busts (and lose) if the value of this card is higher than 3. He wins if the value is 3, or if the value is 2 and the score of both previous players is less than 20. In the remaining situations he is tied with at least one player. His chances are:

   - win: 0.202
   - win together with one player: 0.586
   - all 3 players tied: 0.515
   - lose: 0.688

   So, in the last case the chances are better. If any of the previous players does not stand on 17, the difference between these chances is even bigger.

   Conclusion: Although he scored 18, he should take another card!

2. *Open game with 8 card decks. Two players are playing, and the first of them has 10 and 6. What should he do?*

   Assume that the second player never stands before 18. If the first player stands his chances are:

   - win: 0.45
   - lose: 0.55

   If he take another card we get the following probabilities:

   - win: 0.357
   - tied: 0.067
   - lose: 0.576

   So, the first player has better chances if he stands on 16! But, assume now that the second player always stands on 17. In the similar way we get the following results. If the first player stands, his chances are:

   - win: 0.355
   - lose: 0.645

   If the first player take another card his chances are:

- win: 0.385
- tied: 0.061
- lose: 0.554

So in this case he has better chances if he take another card.

Conclusion: The answer depends on the standing strategy of the second player. We conclude earlier that "stand on 18" is a better strategy for him. But this situation shows that this strategy is not always the best choice. We remind that the second player is choosing his strategy when he does not know whether the first player has the same strategy in every round or not. An additional conclusion would be that the standing strategy should be changed through the game.

3. *Dealer game with 8 card decks. The player has 9 and 8. Discuss what should he do if the game is playing under the rules described in (ii) or (iii) (see the previous section).*

   In (ii), if he stands his winning probability is 0.516 (better chances than the dealer), and if he take another card his winning probability is 0.437.

   In (iii), if he stands his winning probability is 0.45, and if he take another card his winning probability is 0.427.

   Conclusion: In both situations he should stand.

4. *Dealer game described in (ii). The player and dealer stand, but the player has less cards in the hand. Compute the probability of the player's win.*

   If the player always stands on 17 then, according to Table 5, he has over than 50 percent chances to win. For example, if he took 2, and the dealer took 3 cards, then the player's winning probability is 0.629, etc. The other player's standing strategy is left to the reader.

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